Permeability estimation using vortex-based method and synthetic samples from X-ray micro-CT

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1. Introduction

The investigation of fluid flow at the pore scale, based on X-ray scans of a porous medium of interest, is one of the most challenging problems in CFD. However, the predictive power of many existing simulators remains to be demonstrated. Model validation is crucial, as direct observation of the flow field inside real porous media is generally not possible, and since model and experiment generally exhibit differences in geometry, boundary conditions and/or physics. In this study, we present the cross-validation of a new flow simulator from a numerical point of view via a permeability estimation. The pore scale geometry used in the computation is obtained using X-ray tomography.

2. A next-generation hybrid grid-particle solver for pore-scale modeling

Flow simulation at pore scale is difficult because of the inherent complexity of the geometry, which includes a fluid-solid interface with possible roughness. The high resolution needed to capture relevant geometrical details has to be handled without using tremendous memory resources, excluding traditional assembling methods like finite elements or finite volumes. In our study, we use a robust hybrid grid-particle method [1, 2] to solve the advection-diffusion problem. This complete problem writes in the domain Ω

$$\begin{cases} u = \bar{u} \text{ in solid domain,} \\ -\mathrm{div}(2\mu(\alpha, u)D(u)) = f - \nabla P \text{ in fluid domain ,} \\ \mathrm{div} u = 0 \text{ in } \Omega, \\ \partial_t \alpha + u \cdot \nabla \alpha - \eta \Delta \alpha = 0 \text{ in } \Omega, \end{cases} \tag{1}$$

where u is the fluid velocity, $D(u) = (\nabla u + \nabla u^T)/2$, μ is the fluid viscosity, f is the external force, P is the pressure, α is the transported quantity and η is its diffusion coefficient. The fluid velocity field is first computed on a Cartesian grid, then particles manage transport. Interpolation between grid and (moving) particles needs to conserve mass and positivity for the sake of physical coherence. Such coupled diffusion-transport model applies to heterogeneous fluids [6] and viscous mixtures [5]. Velocity field computation relies on the solution of the 3D Stokes equation in its penalized formulation [3]. In our solver, this is carried out using an adapted velocity-vorticity formulation based on the method described in [4] for external flows. This allows to efficiently satisfy adherence and slipping conditions at fluid-solid interface.

3. Simulation setup and results

An important point is the validation of the model by assessing its performances via direct measurements. To assess the capability to numerically predict the permeability of a porous medium, a dedicated experimental setup has been built. For each studied configuration, a 3D tomographic scan is made from the dry state. The reconstructed geometry is extracted and the resulting 3D geometrical virtual volume is used as input for numerical computation.

Different configurations are tested: first, a network of smooth micro-spheres, packed in the cylindrical cavity of a Hassler-like cell, is used for validation. This configuration of interest consists of a pack of smooth micro-spheres with nominal diameter equal to $553 \pm 11\ \mu m$. This diameter was selected to have around 10 spheres across the diameter of the test cell. The region of interest will be around 5003 voxels, which can be handled by a typical numerical solver, and each sphere is described by around 50 voxels across its diameter, which is sufficient to have an accurate geometrical representation. The voxel size is set to $15\mu m$. This sample permits the validation of an accurate 3D model, as it presents a complex topology. While the micro-spheres uses a synthetic medium, a second test can be set which relies on a natural geometry. A rock sample, precisely a Bentheimer stone (a well-documented sandstone), is chosen to conduct this real case test. A rendering of the fluid-solid splitting for this geometry is shown in figure 1.

Figure 2 depicts a rendering of the resulting velocity field computing over the domain, while figure 3 shows particle advection according to the computed field. Measurements using different fluids, using multi-modal sphere-packs and at higher spatial resolutions are foreseen as well. In order to validate the numerical process, the solution fields are processed to get the permeability of the sample via the averaged Darcy law. The numerical permeability values are finally compared to experimental ones on the same samples.

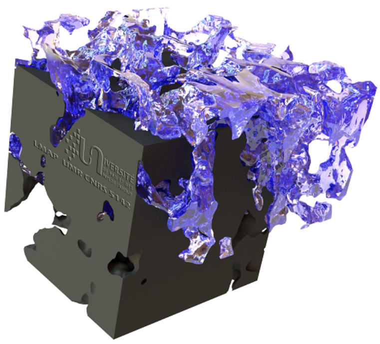

Figure 1: Rendering of fluid and solid phase inside a 299^3 Bentheimer sandstone

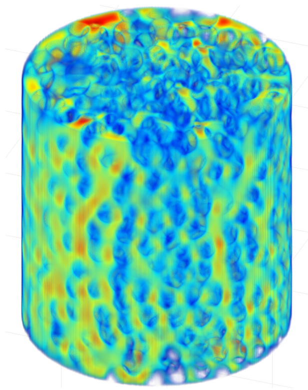

Figure 2: Norm of velocity field inside beads network (257^3 grid)

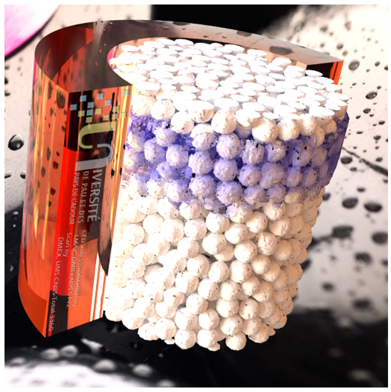

Figure 3: Rendering of advected particles from computed velocity field

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